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Recherches sur l'oeuvre mathématique de Mei Wending (1633–1721). By Jean-Claude Martzloff. Mémoires de l'Institut des Hautes Etudes Chinoises, Vol. 16. Paris (Collège de France). 1981. 472 pp.

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After reaching its apogee in the works of the great Song and Yuan mathematicians of the late 13th and early 14th centuries, Chinese traditional mathematics fell into decline and lay almost forgotten for more than three centuries. Mei Wending has long been recognized as one of the chief figures in the revival of interest in traditional Chinese mathematics which came about under the impact of Western mathematics, introduced into China during the 16th and 17th centuries. Few studies of his work have been made by Western writers, and this is the first major study in a European language to deal with his mathematical thought in some detail.

Martzloff notes that Hashimoto Keizo [1970, 1973], in trying to show that Mei Wending was interested, not so much in developing the new mathematical ideas from the West as in integrating them with Chinese ideas into a single whole, limits his investigations to Mei Wending's ideas on "geometry" and "arithmetic." Before coming to any conclusion, Martzloff suggests that we need to clarify exactly what is meant by the terms "geometry" and "arithmetic" in the earlier Chinese context, that is to say, what kind of problems, constructions, and reasoning the Chinese studied under these two headings; in what way the ideas of Chinese mathematics differed from those of European mathematics as perceived through the Chinese translations of Western works of the time; and how these ideas were organized, and what was their internal logic.

Like all traditional Chinese mathematical texts, Mei Wending's mathematics was expressed in rhetorical form. While translations of ancient texts into modern algebraic symbolism enable us to discover the mathematical results therein, they cannot always reveal to us their underlying logic, which frequently rests on mathematical ideas quite different from our own. Such translations tend, therefore, to erase the differences between ancient and modern mathematics and conceal the internal logic on which ancient mathematics is built, reducing all ancient mathematics to mere anticipations of modern mathematics. Moreover, in the case of

China, the ideographic nature of the Chinese written language means that translation of Chinese rhetoric into rhetorical French would give only a very inadequate idea of the real nature of Chinese mathematics. To overcome these difficulties, Martzloff uses modern symbolism simply as a means for making his explanations readily comprehensible to modern readers. At the same time, in order to help them rediscover the internal logic of the Chinese text, he not only develops the "programmatic" style of translation proposed by Hoe [1977, 1978], but also includes "visual" translations in the form of a sequence of geometrical figures. The original Chinese text, with vocabulary notes and other explanations added, is given with every translation, so that even readers with only limited Chinese can, if they wish, check the accuracy of the translations and interpretations proposed.

To give an idea of the state of mathematics in China in the 17th century, by which time many of the earlier works of traditional Chinese mathematics had been lost and forgotten or were no longer understood, Martzloff lists the traditional Chinese works and the Chinese adaptations of European works which Mei Wending had studied, and on the basis of which he attempted his synthesis of East and West. This is followed by a summary of the contents of each chapter of thirteen of Mei Wending's most important mathematical works, after which Martzloff proceeds to a detailed analysis of Mei Wending's arithmetic, his methods of numerical analysis, and geometry.

A decimal place-value system of numeration has existed in China since earliest times. Actual computation was done with counting-rods until the 14th century, after which time rod-computation was gradually replaced by computations on the abacus. Both methods used a decimal system in which the place-value was determined by designating a particular column of rods or beads as the units' column. Mei Wending's achievement was to adapt Western methods of written calculation to Chinese ideographic writing. The advantage of this over traditional Chinese methods lay not in the greater efficiency of written calculations, which is debatable, but in their being verifiable at any moment. In computations with rods or with beads, as with modern hand-held calculators, the intermediate calculations are often irretrievably lost, so that the checking of complex calculations can become very tedious, hence the appeal of the *gelosia* method of multiplication and the galley method of division to Chinese mathematicians of Mei Wending's time, and to school teachers of today. Moreover, not only does Mei Wending explain how to use written calculations to carry out the four operations of arithmetic on integers and fractions as well as other techniques, but, in view of the importance accorded to calculations with instruments in traditional Chinese mathematics, he also gives adaptations for the Chinese use of Napier's rods and of Galileo's proportional compass.

The major part of Martzloff's study is concerned with Mei Wending's techniques of numerical analysis, such as the extraction of nth roots, the numerical solution of quadratic and cubic equations, the solution of systems of n linear equations in n unknowns, and the interpolation of series. It is in this part of the study, and in the succeeding section on geometry, that we can see most clearly the advantages of Martzloff's method of translation and explication for revealing the way in which Chinese mathematicians reasoned before the advent of modern algebraic symbolism.

The technique for extracting *n*th roots was not new in China, but the ancient works known to Mei Wending either stopped at fourth roots or described methods which were impractical. Without the help of modern symbolism, Mei Wending was able to develop a general method based on the coefficients of the "Pascal" triangle. Although Mei Wending's method was not rigorous, Martzloff reminds us that lack of rigor in mathematical thinking in premodern days was not a characteristic peculiar to the Chinese. His desire to generalize the methods for finding square and cubic roots to *n*th roots marks Mei Wending as a true mathematician, and indeed, Martzloff gives other examples that indicate that Mei Wending was more concerned with deepening the principles of mathematics by generalizing its methods than with developing algorithms for concrete application. It was through reading a Chinese adaptation of a European work and perceiving what he thought to be its mistakes and inadequacies that Mei Wending was led to study the principles of Chinese mathematics more closely.

By translating the Chinese phrases which correspond to solving a second- or third-degree equation as taking a "rectangular" or "parallelepipedic" root, respectively, Martzloff is able to give us some idea of how Chinese mathematicians in general, and Mei Wending in particular, thought of these processes, namely, as generalizations of the taking of square and cubic roots, respectively. However, Mei Wending's geometrical method was difficult to generalize since equations of degree higher than three would have required the representation of hypervolumes. Chinese mathematicians of the Song and Yuan dynasties, for their part, used "Horner's" method for solving higher order polynomial equations with numerical coefficients [Mikami 1913; Wang Ling & Needham 1955; Needham 1959; Librecht 1973; Hoe 1977; Lam Lay Yong 1982), and had no need for such figures.

Problems of second and third degree already had appeared in China in the Han dynasty, although the earliest extant text to give the actual steps in the solution of such problems dates only from the 13th century [Lam Lay Yong 1978]. The Song and Yuan mathematicians were able to solve not only higher order polynomial equations, but even certain systems of polynomial equations in several variables [Hoe 1977]. Their works, lost in China, had by the 17th century sunk into oblivion, but reaching Japan via Korea, gave rise to developments of exceptional depth and richness.

Problems that lead to the solution of *n* linear equations in *n* unknowns are grouped by Mei Wending under the generic title of *fangcheng*. The technique of solution, equivalent to the reduction of the matrix of coefficients to triangular form, had been known in China a millenium earlier, but by Mei Wending's time, the books available were full of mistakes in understanding and actual errors. Instead of trying to correct these, Mei Wending concentrated on trying to reconstruct the method and clarify its underlying theory. In doing so, it was impossible for him to appeal to European works, since they were silent on the matter, and it was only after twenty years' work that he felt confident enough to publish his results.

He first borrows the Confucian idea of the "rectification of names" to define precisely the meaning of the fundamental technical expressions of the method, and then shows how a problem stated in words is put into numerical form, that is, into an array of positive numbers, negative numbers, and "empty spaces" or zeros. China is the most ancient civilization to have conceived the idea of positive and negative quantities, which in the Han dynasty were represented by red and black rods respectively, whereas under the Song, a stroke was placed through the last figure of a number to indicate that it was negative. By the 17th century, these techniques had been forgotten, and so Mei Wending simply places the ideograph for positive or for negative before the number in question. The use of an empty space to indicate a zero is a natural consequence of computations with rods or beads.

Following conversion of the problem into a numerical array, the solution is obtained, as already mentioned, by what we would think of today as a triangular reduction of the matrix of the system. To solve systems of two or three linear equations, European mathematics of the same period resorted to the rule of false position or of double-false position, respectively. Neither the Chinese nor the European methods made use of algebraic symbolism. The Chinese reduction method does not seem to have appeared in Europe before Gauss (1777–1855), but was well known in the various works that serve as landmarks in the history of Chinese mathematics [Libbrecht 1973]. There is no question today of the greater efficiency of the *fangcheng* method as compared with the European methods of Mei Wending's time.

Thus Mei Wending's innovatory role lies not in the invention of a new method, but in the methodical and systematic reconstruction of often very brief or even missing texts. The method makes clear the resolutely numerical approach of Chinese mathematics in seeking efficient numerical solutions, rather than in developing theoretical solutions which, however elegant, may lead to quite impractical computational techniques, as does Cramer's rule, for example. This approach is seen also in the techniques for interpolation, intended for elaborating tables for predicting the positions of the sun, moon, and five planets at any moment of the year. At first, second-degree polynomials were used, and it was only in the 13th century that recourse was had to third-degree polynomials. Mei Wending participated in the compilation of the chapters in the Ming dynasty annals that deal with these subjects.

Chinese geometry laid great stress on the theory of the right-angled triangle, developing from it not only the theorem of Pythagoras but also relations between geometrical areas which we would express today by algebraic formulae. Van der Waerden [1976] thinks that geometrical figures in ancient mathematics served to illustrate preexisting and already developed ideas. Martzloff suggests, on the other hand, that in China the development of algebraic ideas and techniques came from geometrical considerations. Since the works of the Song and Yuan mathematicians had not yet been recovered in his day, Mei Wending's theory of the right-angled triangle does not differ greatly from that developed in China a thousand years earlier. He reinterpreted a considerable part of Euclidean geometry in terms of this theory, and in so doing, was able to obtain other Euclidean theorems which were still unknown in China at that time. He did so without using any part of the axiomatic-deductive system on which Euclidean geometry rests, for Chinese geometry was different from that of the Greeks both in its aims and its methods. Moreover, Martzloff also suggests that for a 17th century Chinese mathematician to study logic by reading Euclid would have been as difficult as for a modern reader to learn elementary mathematics by studying Bourbaki. Formal logic in the West springs from the philosophy of the sophists, whereas in China the ancient sophists had little influence on the development of logic. Mei Wending relies therefore on visual intuition, and sees no need to prove the "obvious," or to establish existence theorems.

Mei Wending's proofs, being based on the right-angled triangle, were concerned with the metric aspect of geometry, and depend on a method of equidecomposability in which a geometrical figure is decomposed into a number of constituent parts from which a new figure of equal area is reconstructed. It is a method which dates back to the Han period, and enabled Chinese mathematicians to obtain results not only on areas but also on the extraction of square and cubic roots, the solution of second- and third-degree equations, and so on, giving rise to a geometrical algebra. Properties such as parallelism, perpendicularity, and similarity were simply accepted a priori. To show a Western reader how the method works, Martzloff here makes very effective use of "visual" translations in addition to his "programmatic" translations, analyzing in detail not only Mei Wending's ideas on plane and solid geometry but also his results on series. While such visual arguments are basically heuristic, they give us a good idea of the difficulties which confronted Chinese 17th century mathematicians.

Although proofs were not unknown before Mei Wending's time, Martzloff argues that in his concern for the justification of his mathematical results, Mei Wending's work marks a radical break with the majority of his predecessors, who were usually interested only in algorithms and end results. His conceptions led to a renewal of interest by Chinese mathematicians in speculative mathematics, and this was not wholly unconnected with the introduction of Western science into China. Mei Wending made a critical synthesis of Western and Chinese ideas, but was hampered by the low level of the Western works accessible to him in Chinese translation and of the Chinese works available at that time. He was also limited by his rejection of the fundamental axiomatic-deductive reasoning of the West, and by his acceptance of the idea that concrete reality is the only basis from which mathematical concepts can be developed. Nevertheless, he succeeded in building a coherent system of mathematics, guided by two of the principal streams of the Chinese tradition: linearity in numerical analysis and equidecomposability in geometry.

Martzloff's study, of which a brief résumé has been presented above, is a model of lucidity. Mathematicians of the 19th century praised Mei Wending as the outstanding Chinese mathematician of the Qing period. In part, this was for his brilliance as a teacher, a brilliance marked by the clarity and the detail of his explanations. In this respect, Martzloff is a worthy follower in his footsteps. His study will be invaluable to anyone investigating the history of Chinese mathematics, and in particular the strengths and weaknesses of the theoretical foundations of Chinese mathematical thought. It can also be read with profit by anyone interested in the differences between Chinese and European views of mathematics, and in the way in which ideas change in their transmission from one culture to another.

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